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Research Article

Beamforming-Based Physical Layer Network Coding for Non-Regenerative Multi-Way Relaying

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We propose non-regenerative multi-way relaying where a half-duplex multi-antenna relay station (RS) assists multiple single-antenna nodes to communicate with each other. The required number of communication phases is equal to the number of the nodes, N. There are only one multiple-access phase, where the nodes transmit simultaneously to the RS, and $N-1$ broadcast (BC) phases. Two transmission methods for the BC phases are proposed, namely, multiplexing transmission and analog network coded transmission. The latter is a cooperation method between the RS and the nodes to manage the interference in the network. Assuming that perfect channel state information is available, the RS performs transceive beamforming to the received signals and transmits simultaneously to all nodes in each BC phase. We address the optimum transceive beamforming maximising the sum rate of non-regenerative multi-way relaying. Due to the nonconvexity of the optimization problem, we propose suboptimum but practical signal processing schemes. For multiplexing transmission, we propose suboptimum schemes based on zero forcing, minimising the mean square error, and maximising the signal to noise ratio. For analog network coded transmission, we propose suboptimum schemes based on matched filtering and semidefinite relaxation of maximising the minimum signal to noise ratio. It is shown that analog network coded transmission outperforms multiplexing transmission.

1. Introduction

The bidirectional communication channel between two nodes was introduced in [1]. Recently, as relay communication becomes an interesting topic of research, the work in [1] was extended by other works, for example, those in [2–7], for bidirectional communication using a half-duplex relay station (RS).

Bidirectional communication using a half-duplex RS can be realised in 4-phase [2, 8], 3-phase [9–11], or 2-phase communication [2, 7, 8]. The latter was introduced as two-way relaying protocol in [2], which outperforms the 4-phase (one-way relaying) communication in terms of the sum rate performance. This is due to the fact that two-way relaying uses the resources more efficiently. In two-way relaying, the two communicating nodes send their data streams simultaneously to the RS in the first communication phase, the multiple-access (MAC) phase. In the second phase, the broadcast (BC) phase, the RS sends the superposition of the nodes' data streams to the nodes. After applying self-interference cancellation, each node obtains its partner's data streams. Two-way relaying adopts the idea of network coding [12], where the RS uses either analog network coding [2–4] or digital network coding [2, 5–7].

An RS that applies analog network coding can be classified as a non-regenerative RS since the RS does not regenerate (decode and re-encode) the data streams of the nodes. A non-regenerative RS has three advantages: no decoding error propagation, no delay due to decoding and deinterleaving, and transparency to the modulation and coding schemes being used at the nodes [8]. Non-regenerative, in general, may be, for example, amplify-and-forward in strict sense, that is, pure amplification of the received signal [2], beamforming [8], or compress-and-forward [13]. In this paper, we consider a non-regenerative relaying where the RS performs transceive beamforming.

It is widely known from many publications, for example, [14, 15], that the use of multiple antennas improves the spectral efficiency and/or the reliability of the communication systems. A multi-antenna RS, which serves one bidirectional pair using two-way relaying, is considered in [16–18] for a regenerative RS and in [8, 19, 20] for a non-regenerative RS. For the non-regenerative case, while [8, 19] assume multi-antenna nodes, [20] assumes single-antenna nodes. Their works consider optimal beamforming maximising the sum rate as well as linear transceive beamforming based on Zero Forcing (ZF) and Minimum Mean Square Error (MMSE), and in [8] also Maximisation of Signal to Noise Ratio (MSNR) criteria.

Multi-user two-way relaying, where an RS serves more than one bidirectional pair, is treated in [21–23] for a regenerative RS and in [24, 25] for a non-regenerative RS. In [21], all bidirectional pairs are separated using Code Division Multiple Access. Every two nodes in a bidirectional pair have their own code which is different from the other pairs' codes. In contrast to [21], in [22, 23], the separation of the pairs in the second phase is done spatially using transmit beamforming employed at the RS. For the non-regenerative case, the multi-antenna RS performs transceive beamforming to separate the nodes [24] or the pairs [25]. In [24], ZF and MMSE transceive beamforming for multi-user two-way relaying is designed and the bit error rate performance is considered. Different to [24], in [25] pair-aware transceive beamforming is performed at the RS. The RS separates only the data streams from different pairs and, thus, each node has to perform self-interference cancellation. The sum rate performance is considered and it is shown that the pair-aware transceive beamforming outperforms the ZF one. Additionally, [25] addresses the optimum transceive beamforming maximising the sum rate of the non-regenerative multi-user two-way relaying.

In recent years, applications such as video conference and multi-player gaming are becoming more popular. In such applications, multiple nodes are communicating with each other. An N-node multi-way channel is one in which each node has a message and wants to decode the messages from all other nodes [26]. Until now, there are only few works on such a multi-way channel, for example, the work of [26, 27], where [1] is a special case when the number N of the nodes is equal to two.

A multi-way relay channel, where multiple nodes can communicate with each other only through an RS, is considered in [28]. A full-duplex communication, where full-duplex nodes communicate with each other through a full-duplex RS, is assumed. However, full-duplex nodes and relays are still far from practicality and half-duplex nodes and relays are more realistic [2, 29]. Therefore, efficient communication protocols to perform multi-way communication between half-duplex nodes with the assistance of a half-duplex RS are needed.

In multi-way communication, if all N nodes are half-duplex and there are direct links between them, the required number of communication phases in order for each node to obtain the information from all other nodes is N, as depicted in Figure 1(a) for the case of $N = 3$, namely, nodes S0, S1 and S2. Assuming that there are no direct links between the nodes, and that they communicate only through the assistance of an RS, if the RS applies the one-way relaying protocol, the required number of phases is $2N$, as shown in Figure 1(b) for the case of $N = 3$.

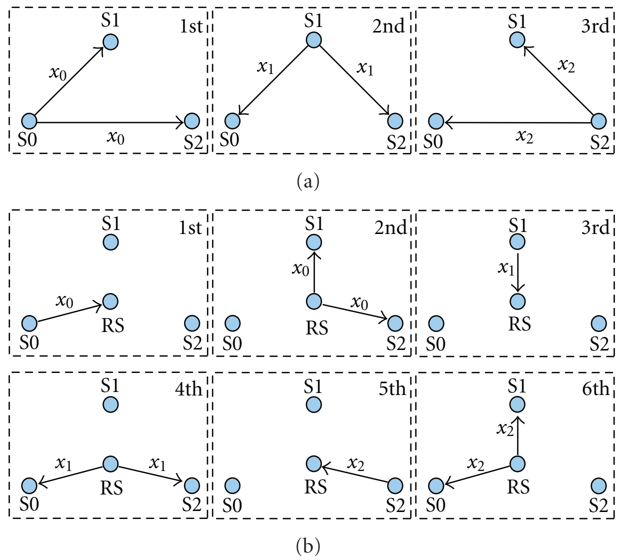

Figure 1: Multi-way communication: (a) with direct link; (b) with the assistance of a relay station using the one-way relaying protocol.

Recently, the authors of this paper proposed a multi-way relaying protocol where a half-duplex regenerative RS assists multiple half-duplex nodes to communicate with each other in [30]. A transceive strategy which ensures that the RS is able to transmit with the achievable MAC rate while minimising the transmit power is proposed. The required number of communication phases for the multi-way relaying is only N.

Different to [30], in this paper, we propose non-regenerative multi-way relaying where the required number of phases is also N. There is only one MAC phase, where all nodes transmit simultaneously to the RS and there are $N-1$ BC phase, where the RS transmits to the nodes. The RS is equipped with multiple antennas to spatially separate the signals received from and transmitted to all nodes. Our work is a generalisation of the non-regenerative two-way relaying; that is, if $N = 2$, we have the non-regenerative two-way relaying case.

In this paper, we propose two different transmission methods for the BC phases, namely, multiplexing transmission and analog network coded transmission. Using multiplexing transmission, in each BC phase, the RS spatially separates the data streams received from the nodes and transmits a different data stream to each node. On the other hand, using analog network coded transmission, the RS superposes two out of N data streams and simultaneously transmits the superposed data stream to the nodes. Prior to decoding, each node has to perform self- and known-interference cancellation. This is a cooperation method between the RS and the nodes to manage the interference in the network, which improves the performance in the network.

It is assumed in this paper that perfect channel state information (CSI) is available, such that the multi-antenna RS can perform transceive beamforming. We first derive the achievable sum rate and then address the optimum transceive beamforming maximising the sum rate of non-regenerative multi-way relaying. Because the optimisation problem is nonconvex, it is too complex to find the optimum solution. Therefore, we propose suboptimum but practical signal processing schemes at the RS, namely, suboptimum Spatial Multiplexing Transceive Beamforming (SMTB) schemes for multiplexing transmission and suboptimum Analog Network Coding Transceive Beamforming (ANCTB) schemes, which are specially designed for analog network coded transmission. Three suboptimum SMTB algorithms are designed, namely, Zero Forcing (ZF), Minimum Mean Square Error (MMSE) and Maximisation of Signal to Noise Ratio (MSNR). Two suboptimum ANCTB algorithms are designed, namely, Matched Filter (MF) and semidefinite relaxation (SDR), which is based on the semidefinite relaxation of maximising the minimum signal to noise ratio problem. The performances of these schemes are analysed and compared.

This paper is organised as follows. Section 2 explains the protocol and the transmission methods. The system model is provided in Section 3. Section 4 explains the achievable sum rate. Section 5 describes the transceive beamforming. Section 6 provides the performance analysis. Section 7 concludes the work.

Notations. Boldface lower- and upper-case letters denote vectors and matrices, respectively, while normal letters denote scalar values. The superscripts $(\cdot)^{\mathrm{T}}$, $(\cdot)^*$, and $(\cdot)^{\mathrm{H}}$ stand for matrix or vector transpose, complex conjugate, and complex conjugate transpose, respectively. The operators $\mathrm{mod}_N(x)$, $\mathrm{E}\{\mathbf{X}\}$ and $\mathrm{tr}\{\mathbf{X}\}$ denote the modulo N of x, the expectation and the trace of $\mathbf{X}$, respectively, and $\mathcal{CN}(0,\sigma^2)$ denotes the circularly symmetric zero-mean complex normal distribution with variance σ^2.

2. Protocol and Transmission Methods

In this section, the communication protocol and the transmission methods for N-phase non-regenerative multi-way relaying are described. We first explain the protocol for multiplexing transmission followed by the explanation of the protocol for analog network coded transmission.

2.1. Multiplexing Transmission. In N-phase non-regenerative multi-way relaying with multiplexing transmission, in the first phase, the MAC phase, all N nodes transmit simultaneously to the RS. The following $N-1$ phases are the BC phases where the RS transmits to all nodes simultaneously. Using multiplexing transmission, in each BC phase, the RS transmits N data streams simultaneously to all nodes, one data stream for each node. For that purpose, the RS separates the received data stream spatially and in each BC phase transmits to each node one data stream from one of the other $N-1$ nodes. In each BC phase, each node receives a different data stream from a different node, in such a way that after $N-1$ BC phases, each node receives the $N-1$ data streams from the other $N-1$ nodes.

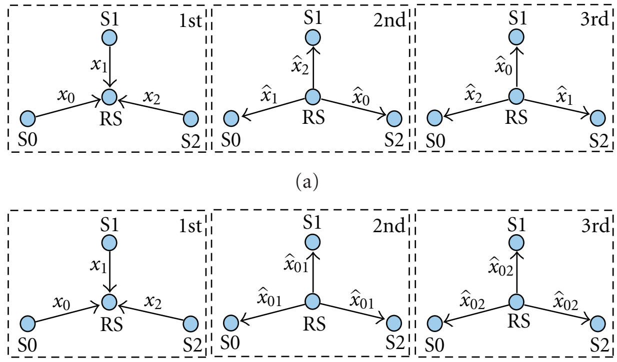

Figure 2: Multi-way relaying: (a) multiplexing transmission; (b) analog network coded transmission.

Figure 2(a) shows an example when three nodes communicate with each other with the help of an RS. In the first phase, S0 sends x_0, S1 sends x_1 and S2 sends x_2 simultaneously to the RS. The RS performs transceive beamforming to spatially separate the data streams. As a result, $\hat{x}_i$ is obtained as the output of the transceive beamforming at the RS, which is the data stream from node i plus the RS's noise and depends on the employed transceive beamforming. In the second phase, the RS forwards $\hat{x}_0$ to S2, $\hat{x}_1$ to S0 and $\hat{x}_2$ to S1. In the third phase, the RS forwards $\hat{x}_0$ to S1, $\hat{x}_1$ to S2 and $\hat{x}_2$ to S0. After completing these three communication phases, each node receives the data streams from all other nodes.

2.2. Analog Network Coded Transmission. As for multiplexing transmission, N-phase non-regenerative multi-way relaying with analog network coded transmission also consists of one MAC phase and $N-1$ BC phases. However, instead of spatially separating each data stream received from and transmitted to the nodes, using analog network coded transmission, in each BC phase the RS superposes two data streams out of the N data streams. The two data streams to be superposed are changed in each BC phase, in such a way that after $N-1$ BC phases, each node receives $N-1$ superposed data streams which contain the $N-1$ data streams from the other $N-1$ nodes. In each BC phase, the superposed data stream is then transmitted simultaneously to the nodes. Therefore, there is no interstream interference as in the case of multiplexing transmission. Consequently, each node has to perform interference cancellation.

Figure 2(b) shows an example of non-regenerative multi-way relaying with analog network coded transmission for the case of $N = 3$. In the first phase, all nodes transmit simultaneously to the RS, S0 sends x_0, S1 sends x_1 and S2 sends x_2. In the second phase, the RS sends $\hat{x}_{01}$ to all nodes. The transmitted data stream $\hat{x}_{01}$ is a superposition of the data streams from S0 and S1 plus the RS's noise. Both S0 and

S1 perform self-interference cancellation, so that S0 obtains x_1 and S1 obtains x_0. Node S2 cannot yet perform self-interference cancellation, since $\hat{x}_{01}$ does not contain its data stream. In the third phase, the RS transmits $\hat{x}_{02}$ to all nodes. Both nodes S0 and S2 perform self-interference cancellation so that S0 obtains x_2 and S2 obtains x_0. Since S1 knows x_0 from the second phase, it performs known-interference cancellation to obtain x_2 in the third phase. For S2, since it knows x_0 from the third phase, it obtains x_1 by performing known-interference cancellation to the received data stream $\hat{x}_{01}$ in the second phase. Thus, S2 needs to wait until it receives the data stream containing its own data stream. After performing self-interference cancellation, it performs known-interference cancellation to obtain the other data stream. After three phases, all nodes obtain the data streams from all other nodes.

Non-regenerative multi-way relaying with analog network coded transmission is a cooperation between the RS and the nodes to manage the interference in the network. Since the nodes can perform the self- and known-interference cancellations, the RS does not need to suppress interference signals which can be canceled at the nodes. Thus, there is no unnecessary loss of degrees of freedom at the RS to cancel those interference signals. Hence, it can be expected that there is a performance improvement when using analog network coded transmission compared to multiplexing transmission.

3. System Model

In this section, the system model of non-regenerative multi-way relaying is described. There are N single-antenna nodes which want to communicate with each other through a multi-antenna RS with M antenna elements. It is assumed that perfect CSI is available so that the RS can employ transceive beamforming. Although in this paper we only consider single-antenna nodes, our work can be readily extended to the case of multi-antenna nodes. We first describe the overall system model for non-regenerative multi-way relaying. Afterwards, we explain the specific parameters required for each of the two transmission methods: multiplexing transmission and analog network coded transmission.

In the following, let $\mathbf{H} \in \mathbb{C}^{M\times N} = [\mathbf{h}_0, \ldots, \mathbf{h}_{N-1}]$ denote the overall channel matrix, with $\mathbf{h}_i \in \mathbb{C}^{M\times 1} = (h_{i,1}, \ldots, h_{i,M})^{\mathrm{T}}, i \in \mathcal{I}, \mathcal{I} = \{0, \ldots, N-1\}$, being the channel vector between node i and the RS. The channel coefficient $h_{i,m}, m \in \mathcal{M}, \mathcal{M} = \{1, \ldots, M\}$, follows $\mathcal{CN}(0, \sigma_h^2)$. The vector $\mathbf{x} \in \mathbb{C}^{N\times 1}$ denotes the vector of $(x_0, \ldots, x_{N-1})^{\mathrm{T}}$, with x_i being the signal of node i which follows $\mathcal{CN}(0, \sigma_x^2)$. The additive white Gaussian noise (AWGN) vector at the RS is denoted as $\mathbf{z}_{\mathrm{RS}} \in \mathbb{C}^{M\times 1} = (z_{\mathrm{RS}1}, \ldots, z_{\mathrm{RS}M})^{\mathrm{T}}$, where $z_{\mathrm{RS}m}$ follows $\mathcal{CN}(0, \sigma_{z_{\mathrm{RS}}}^2)$. It is assumed that all nodes have fixed and equal transmit power.

In non-regenerative multi-way relaying, in the first phase, the MAC phase, all nodes transmit simultaneously to the RS. The received signal at the RS is given by

$$\mathbf{r}_{\mathrm{RS}} = \mathbf{H}\mathbf{x} + \mathbf{z}_{\mathrm{RS}}. \tag{1}$$

The non-regenerative RS performs transceive beamforming to the received signals and transmits to the nodes simultaneously. We assume that in each BC phase the RS transmits with power q_{RS}. Assuming reciprocal and stationary channels in the N phases, the downlink channel from the RS to the nodes is simply the transpose of the uplink channel $\mathbf{H}$.

Let $\mathbf{G}^n$, $n \in \mathcal{N}, \mathcal{N} = \{2, \ldots, N\}$, denote the n-th phase transceive beamforming matrix. The received signal vector of all nodes in the n-th BC phase can be written as

$$\mathbf{y}_{\mathrm{nodes}}^n = \mathbf{H}^{\mathrm{T}}\mathbf{G}^n(\mathbf{H}\mathbf{x} + \mathbf{z}_{\mathrm{RS}}) + \mathbf{z}_{\mathrm{nodes}}, \tag{2}$$

where $\mathbf{z}_{\mathrm{nodes}} = (z_0, \ldots, z_{N-1})^{\mathrm{T}}$ with z_k being the AWGN at a receiving node k which follows $\mathcal{CN}(0, \sigma_{z_k}^2)$. Accordingly, the received signal at node k while receiving the data stream from node i in the n-th BC phase is given by

$$y_{k,i}^n = \underbrace{\mathbf{h}_k^{\mathrm{T}}\mathbf{G}^n\mathbf{h}_i x_i}_{\text{useful signal}} + \underbrace{\sum_{\substack{j=0 \\ j\neq i}}^{N-1} \mathbf{h}_k^{\mathrm{T}}\mathbf{G}^n\mathbf{h}_j x_j}_{\text{interference signals}} + \underbrace{\mathbf{h}_k^{\mathrm{T}}\mathbf{G}^n\mathbf{z}_{\mathrm{RS}}}_{\text{RS's propagated noise}} + z_k. \tag{3}$$

In this paper, we propose multiplexing transmission and analog network coded transmission for non-regenerative multi-way relaying. In the following, we define the relationship of the BC phase index $n, n \in \mathcal{N}$, the receiver index $k, k \in \mathcal{I}$ and the transmitter index $i, i \in \mathcal{I}$, whose data stream shall be decoded in the n-th BC phase by the receiving node k for both transmissions.

Multiplexing Transmission. If the RS is using multiplexing transmission, the relationship is defined by

$$i = \mathrm{mod}_N(k + n - 1), \tag{4}$$

Figure 2(a) shows the example of multiplexing transmission for three nodes.

Analog Network Coded Transmission. If the RS is applying analog network coded transmission, in each BC phase, each node needs to know which data streams from which two nodes have been superposed by the RS. This might increase the signaling in the network. Thus, assuming that each node knows its own and its partners' indices, we propose a method for choosing data streams to be network coded by the RS which does not need any signaling. We choose the data stream from the lowest index node Sv, $v = 0$, and superpose this data stream with one data stream from another node S$w, w \in \mathcal{I} \setminus \{0\}$, which is selected successively based on the relationship defined by $w = n - 1, n \in \mathcal{N}$. In the n-th phase, the RS sends x_{0w} to all nodes simultaneously. Node S$k, k = 0$, receives the data stream from node S$i, i = w$, and it simply performs self-interference cancellation to obtain x_w. The same applies to node S$k, k = w$, it simply performs self-interference cancellation to obtain x_0. Node S0 needs to perform only self-interference cancellation in each BC phase to obtain the other nodes' data streams. The other $N - 1$ nodes S$w, w \in \mathcal{I} \setminus \{0\}$, need to perform self-interference cancellation once they receive the data stream containing their data stream to obtain x_0 and, after knowing x_0, they perform known-interference cancellation by canceling x_0 from each of the received data streams that are received in the other BC phases. Therefore, the relationship can be written as

$$i = \begin{cases} 0, & \text{for } k = n-1, \\ n-1, & \text{otherwise.} \end{cases} \tag{5}$$

Figure 2(b) shows the example of analog network coded transmission for 3 nodes.

Even though x_0 is transmitted $N-1$ times to the nodes, it does not increase the information rate of x_0 at the other $N-1$ nodes. Once x_0 is decoded and known by the nodes, there is no uncertainty of x_0 in the other data streams.

The general rule for the superposition of two data streams in each BC phase is that we have to ensure that the data stream from each node has to be superposed at least once. For $N = 3$, assuming reciprocal and stationary channel in the N phases, there are three options which fulfill the general rule. The first one is as explained above, namely, $\hat{x}_{01}$ and $\hat{x}_{02}$. The other two options are by superposing $\hat{x}_{01}$ and $\hat{x}_{12}$ or by superposing $\hat{x}_{12}$ and $\hat{x}_{02}$. For each of the possible superposition options, exchanging the superposed data streams to be transmitted in the BC phases will result in the same performance due to the assumption of the stationarity of the channel. The higher the N, the more options for superposing the data streams which fulfill the general rule.

4. Achievable Sum Rate

In this section, we explain the achievable sum rate of non-regenerative multi-way relaying. We define the achievable sum rate in the network as the sum of all the rates received at all the nodes. We begin this section with the definition of the Signal to Interference and Noise Ratio (SINR), which is needed to determine the achievable sum rate of non-regenerative multi-way relaying. Afterwards, the achievable sum rate expressions for two different cases, namely, asymmetric and symmetric traffic cases, are given.

4.1. Signal to Interference and Noise Ratio. In this section, we derive the SINR, first for multiplexing transmission and then for analog network coded transmission. For multiplexing transmission, given the received signal in (3), the SINR for the link between receive node Sk and transmit node Si is given by

$$\gamma^n_{\text{mux}_{k,i}} = \frac{S}{I_\text{s} + I_\text{os} + Z_\text{RS} + Z_k}, \tag{6}$$

with the useful signal power

$$S = \mathrm{E}\left\{ \left| \mathbf{h}_k^\mathrm{T} \mathbf{G}^n \mathbf{h}_i x_i \right|^2 \right\} = \left| \mathbf{h}_k^\mathrm{T} \mathbf{G}^n \mathbf{h}_i \right|^2 \sigma_x^2, \tag{7}$$

the self-interference power

$$I_\text{s} = \mathrm{E}\left\{ \left| \mathbf{h}_k^\mathrm{T} \mathbf{G}^n \mathbf{h}_k x_k \right|^2 \right\} = \left| \mathbf{h}_k^\mathrm{T} \mathbf{G}^n \mathbf{h}_k \right|^2 \sigma_x^2, \tag{8}$$

the other-stream interference power

$$I_\text{os} = \sum_{\substack{j=0 \\ j \neq \{k,i\}}}^{N-1} \mathrm{E}\left\{ \left| \mathbf{h}_k^\mathrm{T} \mathbf{G}^n \mathbf{h}_j x_j \right|^2 \right\} = \sum_{\substack{j=0 \\ j \neq \{k,i\}}}^{N-1} \left| \mathbf{h}_k^\mathrm{T} \mathbf{G}^n \mathbf{h}_j \right|^2 \sigma_x^2, \tag{9}$$

the RS's propagated noise power

$$Z_\text{RS} = \mathrm{E}\left\{ \left| \mathbf{h}_k^\mathrm{T} \mathbf{G}^n \mathbf{z}_\text{RS} \right|^2 \right\} = \left| \mathbf{h}_k^\mathrm{T} \mathbf{G}^n \right|^2 \sigma_{z_\text{RS}}^2, \tag{10}$$

and the receiving node k's noise power

$$Z_k = \mathrm{E}\left\{ |z_k|^2 \right\} = \sigma_{z_k}^2. \tag{11}$$

In the n-th BC phase, node k may perform interference cancellation. It subtracts the a priori known self-interference as well as other-stream interference known from the previous BC phases. Once the nodes have decoded other nodes' data streams in the previous BC phases, they may use them to perform known-interference cancellation in a similar fashion to self-interference cancellation. With interference cancellation, the SINR $\gamma^n_{k,i}$ for multiplexing transmission can be rewritten as

$$\gamma^n_{k,i} = \frac{S}{I_\text{notcanc} + Z_\text{RS} + Z_k}, \tag{12}$$

where

$$I_\text{notcanc} = \sum_{\substack{j=0 \\ j \neq k \\ j \notin \mathcal{B}}}^{N-1} \left| \mathbf{h}_k^\mathrm{T} \mathbf{G}^n \mathbf{h}_j \right|^2 \sigma_x^2 \tag{13}$$

is the interference power without self-interference and other-stream interference that have been decoded in the previous BC phases, with $\mathcal{B} = \{ b \mid b = \mathrm{mod}_N(k + o - 1),\ \forall o, o = \{2, \ldots, n-1\}\}$, the set of the nodes whose data streams have been decoded in the previous BC phases.

When the RS is using analog network coded transmission, the SINR is given by

$$\gamma^n_{\text{ANC}_{k,i}} = \frac{S}{I_\text{sok} + Z_\text{RS} + Z_k}, \tag{14}$$

where I_sok is the interference at a receiving node k which can be either self-interference or known interference.

In each BC phase, the RS transmits x_{vw} which is a superposition of the data streams from nodes Sv and Sw. Both nodes Sv and Sw need to perform self-interference cancellation. In this case, the receiving node S$k, k = v$, receives from node S$i, i = w$, and the receiving node S$k, k = w$, receives from node S$i, i = v$. Other nodes which know x_v from the previous BC phase can apply known-interference cancellation to obtain x_w. In this case, the receiving node S$k, k \neq v, k \neq w$, receives the data stream from node S$i, i = w$. Therefore, I_sok is either a self-interference power from (8) or a known-interference power given by

$$I_\text{k} = \mathrm{E}\left\{ \left| \mathbf{h}_k^\mathrm{T} \mathbf{G}^n \mathbf{h}_v x_v \right|^2 \right\} = \left| \mathbf{h}_k^\mathrm{T} \mathbf{G}^n \mathbf{h}_v \right|^2 \sigma_x^2. \tag{15}$$

Since I_{sok} can and should be canceled at each node, the SINR $\gamma_{k,i}^{n}$ for analog network coded transmission with self- and known-interference cancellation is given by

$$\gamma_{k,i}^{n} = \frac{S}{Z_{\text{RS}} + Z_k}. \tag{16}$$

4.2. Sum Rate for Asymmetric Traffic. Given the SINR $\gamma_{k,i}^{n}$ as in Section 4.1, the information rate when node k receives the data stream from node i is given by

$$R_{k,i} = \log_2\left(1 + \gamma_{k,i}^{n}\right). \tag{17}$$

Since all nodes transmit only once, each transmitting node i needs to ensure that its data stream can be decoded correctly by the other $N-1$ receiving nodes $k, k \in \mathcal{I} \setminus \{i\}$. Thus, the information rate transmitted from node i is defined by the weakest link between node i and all other $N-1$ receiving nodes $k, k \in \mathcal{I} \setminus \{i\}$, which can be written as

$$R_i = \min_{k \in \mathcal{I}\setminus\{i\}} R_{k,i}. \tag{18}$$

Finally, the achievable sum rate of non-regenerative multi-way relaying is given by

$$SR_{\text{asym}} = \frac{1}{N}(N-1)\sum_{i=0}^{N-1} R_i. \tag{19}$$

The factor $N-1$ is due to the fact that there are $N-1$ receiving nodes which receive the same data stream from a certain transmitting node i. The scaling factor $1/N$ is due to N channel uses for the overall N communication phases.

One note regarding the achievable sum rate with analog network coded transmission is that, by having (18) for transmitting node Si, $i = v$, we ensure that node Sv transmits x_v with the rate that can be decoded correctly by all other $N-1$ nodes. Thus, having decoded x_v correctly, all other $N-1$ nodes can use it to perform known-interference cancellation in a similar fashion to their self-interference cancellation.

4.3. Sum Rate for Symmetric Traffic. In certain scenarios, there might be a requirement to have a symmetric traffic between all nodes. All nodes communicate with the same data rate defined by the minimum of R_i, $i \in \mathcal{I}$. The achievable sum rate becomes

$$SR_{\text{symm}} = \frac{1}{N}(N-1)N\left(\min_{i\in\mathcal{I}} R_i\right). \tag{20}$$

5. Transceive Beamforming

In this section, the transceive beamforming employed at the RS is explained. It is assumed that the number of antennas at the RS is higher than or equal to the number of nodes, that is, $M \geq N$, since we will derive low complexity linear transceive beamforming algorithms to be employed at RS. In the first subsection, we explain the optimum transceive beamforming maximising the sum rate of non-regenerative multi-way relaying. The following two subsections explain suboptimum but practical transceive beamforming algorithms for both multiplexing and analog network coded transmission.

5.1. Sum Rate Maximisation. In this subsection, the optimum transceive beamforming maximising the sum rate of non-regenerative multi-way relaying for asymmetric traffic is addressed. It is valid for both multiplexing and analog network coded transmissions. Asymmetric traffic is considered since it provides higher sum rate than that symmetric traffic. The optimisation problem for finding the optimum transceive beamforming maximising the sum rate of non-regenerative multi-way relaying for asymmetric traffic can be written as

$$\begin{aligned} &\max_{\mathbf{G}^n} \quad \sum_i \sum_k R_{k,i} \\ &\text{s.t.} \quad \operatorname{tr}\left\{\mathbf{G}^n\left(\mathbf{H}\mathbf{R}_{\mathbf{x}}\mathbf{H}^{\mathrm{H}} + \mathbf{R}_{\mathbf{z}_{\text{RS}}}\right)\mathbf{G}^{n\mathrm{H}}\right\} = q_{\text{RS}}, \end{aligned} \tag{21}$$

where $\mathbf{R}_{\mathbf{z}_{\text{RS}}} = \mathrm{E}\{\mathbf{z}_{\text{RS}}\mathbf{z}_{\text{RS}}^{\mathrm{H}}\}$ is the covariance matrix of the RS's noise, $\mathbf{R}_{\mathbf{x}} = \mathrm{E}\{\mathbf{x}\mathbf{x}^{\mathrm{H}}\}$ is the covariance matrix of the transmitted signal and q_{RS} is the transmit power of the RS.

In this paper, we assume that the transmit power at all nodes is equal and fixed. In order to improve the sum rate, we can have the transmit power at the nodes as variables to be optimised subject to power constraint at each node. However, since there is only one MAC phase, we have to find the optimum transmit power at each node and, simultaneously, the transceive beamforming for all BC phase, $\mathbf{G}^n, \forall n \in \mathcal{N}$. This joint optimisation problem will further increase the computational effort.

The optimisation problem in (21) is nonconvex and it can be awkward and too complex to solve. Thus, in the following subsections we propose suboptimum but practical transceive beamforming algorithms for both multiplexing transmission and analog network coded transmission.

5.2. Suboptimum Spatial Multiplexing Transceive Beamforming. In this subsection, we explain the design of suboptimum Spatial Multiplexing Transceive Beamforming (SMTB) algorithms for multiplexing transmission. We decompose the n-th BC phase transceive beamforming $\mathbf{G}^n$ into receive beamforming $\mathbf{G}_{\text{Rc}}$, permutation matrix $\mathbf{\Pi}^n$ and transmit beamforming $\mathbf{G}_{\text{Tx}}$; that is, $\mathbf{G}^n = \mathbf{G}_{\text{Tx}}\mathbf{\Pi}^n\mathbf{G}_{\text{Rc}}$.

The receive beamforming is only needed to be computed once and can be used for all BC phases' transceive beamforming since there is only one MAC phase. In this paper, we assume reciprocal and stationary channels within the N phases. Therefore, the transmit beamforming should also be computed only once and can be used for all BC phases' transmission. Nevertheless, the transceive beamforming in each BC phase should be different from one BC phase to another, since the RS has to send different data streams to an intended node. In order to define which data stream should be transmitted by the RS to which node in the n-th BC phase, a permutation matrix is used.

The permutation matrix $\mathbf{\Pi}^n$ defines the relationship of receiving index k, the transmitting index i, and the corresponding phase index n. $\mathbf{\Pi}^n$ is given by the operation $\operatorname{colperm}(\mathbf{I}_N, (n-1))$ with $\mathbf{I}_N$, an identity matrix of size N. $\operatorname{colperm}(\mathbf{I}_N, (n-1))$ permutes the columns of the identity matrix $(n-1)$ times circularly to the right. For example, for Figure 2(a), the permutation matrices $\boldsymbol{\Pi}^2 = \begin{pmatrix} 0&1&0\\0&0&1\\1&0&0 \end{pmatrix}$ and $\boldsymbol{\Pi}^3 = \begin{pmatrix} 0&0&1\\1&0&0\\0&1&0 \end{pmatrix}$. Regarding the receive and transmit beamforming, in this paper, we consider three different algorithms, namely ZF, MMSE and MSNR. Receive and transmit beamforming algorithms with those criteria have been derived in [8, 19] for the case of two-way relaying. The optimisation problem with those criteria for multi-way relaying can be written as in [8, 19]. Therefore, in this paper, we use the solution for receive and transmit beamforming from [8, 19] and extend them to suit non-regenerative multi-way relaying by using the permutation matrix as explained above. In the following, we explain the receive and transmit beamforming for the three SMTB algorithms.

5.2.1. Zero Forcing. For multi-way relaying, the minimisation of mean square error subject to the zero forcing constraint can be written as

$$\begin{aligned} \min_{\mathbf{G}^n} \quad & \mathrm{E}\left\{\|\mathbf{x}-\hat{\mathbf{x}}\|^2\right\} \\ \text{s.t.} \quad & \mathrm{tr}\left\{\mathbf{G}^n\left(\mathbf{H}\mathbf{R}_{\mathbf{x}}\mathbf{H}^{\mathrm{H}}+\mathbf{R}_{\mathbf{z}_{\mathrm{RS}}}\right)\mathbf{G}^{n\mathrm{H}}\right\} = q_{\mathrm{RS}}, \\ & \mathbf{x}=\hat{\mathbf{x}}, \quad \text{if } \mathbf{z}_{\mathrm{RS}}=\mathbf{0}, \quad \mathbf{z}_{\mathrm{nodes}}=\mathbf{0}. \end{aligned} \tag{22}$$

The same formulation as in (22) can also be found in [8, 19] for the case of one-way and two-way relaying. In [8, 19] the solution of such a problem is derived.

Using the result from [8, 19], the ZF receive beamforming for multi-way relaying is given by

$$\mathbf{G}_{\mathrm{Rc}} = \left(\mathbf{H}^{\mathrm{H}}\mathbf{R}_{\mathbf{z}_{\mathrm{RS}}}^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{H}}\mathbf{R}_{\mathbf{z}_{\mathrm{RS}}}^{-1} \tag{23}$$

and the ZF transmit beamforming is given by

$$\mathbf{G}_{\mathrm{Tx}} = \frac{1}{p_{\mathrm{ZF}}}\mathbf{H}^{*}\left(\mathbf{H}^{\mathrm{T}}\mathbf{H}^{*}\right)^{-1}, \tag{24}$$

with

$$p_{\mathrm{ZF}} = \sqrt{\frac{\mathrm{tr}\left\{\left(\mathbf{H}^{\mathrm{H}}\boldsymbol{\Upsilon}_{\mathrm{Rc}}^{-1}\mathbf{H}\right)^{-1}\left(\mathbf{H}^{\mathrm{T}}\mathbf{H}^{*}\right)^{-1}\right\}}{q_{\mathrm{RS}}}} \tag{25}$$

and

$$\boldsymbol{\Upsilon}_{\mathrm{Rc}} = \mathbf{H}\mathbf{R}_{\mathbf{x}}\mathbf{H}^{\mathrm{H}} + \mathbf{R}_{\mathbf{z}_{\mathrm{RS}}} \tag{26}$$

5.2.2. Minimum Mean Square Error. For multi-way relaying, the minimisation of mean square error can be written as

$$\begin{aligned} \min_{\mathbf{G}^n} \quad & \mathrm{E}\left\{\|\mathbf{x}-\hat{\mathbf{x}}\|^2\right\} \\ \text{s.t.} \quad & \mathrm{tr}\left\{\mathbf{G}^n\left(\mathbf{H}\mathbf{R}_{\mathbf{x}}\mathbf{H}^{\mathrm{H}}+\mathbf{R}_{\mathbf{z}_{\mathrm{RS}}}\right)\mathbf{G}^{n\mathrm{H}}\right\} = q_{\mathrm{RS}}. \end{aligned} \tag{27}$$

The same formulation as in (27) can also be found in [8, 19] for the case of one-way and two-way relaying. Using the result from [8, 19], the MMSE receive beamforming for multi-way relaying is given by

$$\mathbf{G}_{\mathrm{Rc}} = \mathbf{R}_{\mathbf{x}}\mathbf{H}^{\mathrm{H}}\boldsymbol{\Upsilon}_{\mathrm{Rc}}^{-1}, \tag{28}$$

and the MMSE transmit beamforming is given by

$$\mathbf{G}_{\mathrm{Tx}} = \frac{1}{p_{\mathrm{MMSE}}}\boldsymbol{\Upsilon}_{\mathrm{Tx}}^{-1}\mathbf{H}^{*}, \tag{29}$$

with

$$p_{\mathrm{MMSE}} = \sqrt{\frac{\mathrm{tr}\left\{\mathbf{H}\mathbf{R}_{\mathbf{x}}\mathbf{H}^{\mathrm{T}}\boldsymbol{\Upsilon}_{\mathrm{Tx}}^{-2}\mathbf{H}^{*}\mathbf{R}_{\mathbf{x}}\mathbf{H}^{\mathrm{H}}\boldsymbol{\Upsilon}_{\mathrm{Rc}}^{-1}\right\}}{q_{\mathrm{RS}}}} \tag{30}$$

and

$$\boldsymbol{\Upsilon}_{\mathrm{Tx}} = \mathbf{H}^{*}\mathbf{H}^{\mathrm{T}} + \frac{\mathrm{tr}\left\{\mathbf{R}_{\mathbf{z}_{\mathrm{nodes}}}\right\}}{q_{\mathrm{RS}}}\mathbf{I}_M \tag{31}$$

where $\mathbf{R}_{\mathbf{z}_{\mathrm{nodes}}} = \mathrm{E}\{\mathbf{z}_{\mathrm{nodes}}\mathbf{z}_{\mathrm{nodes}}^{\mathrm{H}}\}$ is the covariance matrix of the noise vector of all nodes.

5.2.3. Maximisation of Signal to Noise Ratio. For multi-way relaying, the maximisation of the signal to noise ratio can be written as

$$\begin{aligned} \min_{\mathbf{G}^n} \quad & \frac{\left|\mathrm{E}\{\mathbf{x}-\hat{\mathbf{x}}\}\right|^2}{\|\mathrm{E}\{\mathbf{x}\}\|_2^2\mathrm{E}\left\{\|\mathbf{H}^{\mathrm{T}}\mathbf{G}^n\mathbf{z}_{\mathrm{RS}}+\mathbf{z}_{\mathrm{nodes}}\|^2\right\}} \\ \text{s.t.} \quad & \mathrm{tr}\left\{\mathbf{G}^n\left(\mathbf{H}\mathbf{R}_{\mathbf{x}}\mathbf{H}^{\mathrm{H}}+\mathbf{R}_{\mathbf{z}_{\mathrm{RS}}}\right)\mathbf{G}^{n\mathrm{H}}\right\} = q_{\mathrm{RS}}. \end{aligned} \tag{32}$$

The same optimisation problem for two-way relaying can be found in [8].

Using the result from [8], the MSNR receive beamforming for multi-way relaying is given by

$$\mathbf{G}_{\mathrm{Rc}} = \mathbf{R}_{\mathbf{x}}\mathbf{H}^{\mathrm{H}}\boldsymbol{\Upsilon}_{\mathrm{Rc}}^{-1}, \tag{33}$$

and the MSNR transmit beamforming is given by

$$\mathbf{G}_{\mathrm{Tx}} = \frac{1}{p_{\mathrm{MSNR}}}\mathbf{H}^{*}, \tag{34}$$

with

$$p_{\mathrm{MSNR}} = \sqrt{\frac{\mathrm{tr}\left\{\mathbf{H}^{*}\mathbf{R}_{\mathbf{x}}\mathbf{H}^{\mathrm{H}}\boldsymbol{\Upsilon}_{\mathrm{Rc}}^{-1}\mathbf{H}\mathbf{R}_{\mathbf{x}}\mathbf{H}^{\mathrm{T}}\right\}}{q_{\mathrm{RS}}}}. \tag{35}$$

5.3. Suboptimum Analog Network Coding Transceive Beamforming. In this subsection, the design of Analog Network Coding Transceive Beamforming (ANCTB) for non-regenerative multi-way relaying is explained. In order to superpose two data streams out of N data streams, the RS has to separate the two data streams from the other received data streams. The superposed data stream needs to be transmitted simultaneously to N nodes. Therefore, we specially design ANCTB to implement analog network coding in non-regenerative multi-way relaying. The proposed ANCTB can be interpreted as a Physical Layer Network Coding (PLNC) for non-regenerative multi-way relaying, where the network coding is performed via beamforming. Thus, the RS does not need to know the modulation constellation and coding which are used by the nodes. This is the difference of the proposed beamforming-based PLNC to the PLNC proposed for two-way relaying in [6, 31].

The n-th BC phase transceive beamforming of ANCTB is decoupled into receive and transmit beamforming. The receive beamforming of ANCTB is basically performing the PLNC by separating two data streams x_v and x_w from the other data streams and superposing them. The receive beamforming is designed based on the ZF Block Diagonalization (ZFBD), which has been proposed in [32] for downlink spatial multiplexing transmit beamforming. Firstly, we use ZFBD to compute the equivalent channel of the two nodes whose data streams will be superposed by the RS. Secondly, we compute the receive beamforming based on the equivalent channel. The superposed data stream needs to be transmitted simultaneously to N nodes. Therefore, we design the transmit beamforming for ANCTB in the same way as designing single-group multicast beamforming. Since we consider reciprocal and stationary channel, the multicast transmit beamforming needs only to be computed once. In the following, we explain the equivalent channel to be used for computing the receive beamforming. Afterwards, the two subsections explain the ANCTB algorithms, that is, Matched Filter and Semidefinite Relaxation, respectively.

Equivalent Channel for Receive Beamforming. In the n-th phase, let $\mathbf{H}_{vw_n}^{\mathrm{T}} \in \mathbb{C}^{2\times M}$ and $\widetilde{\mathbf{H}}_{vw_n}^{\mathrm{T}} \in \mathbb{C}^{(N-2)\times M}$ denote the channel matrix of two nodes Sv and Sw and the channel matrix of the other $N-2$ nodes, respectively. Given the singular value decomposition

$$\widetilde{\mathbf{H}}_{vw_n}^{\mathrm{T}} = \widetilde{\mathbf{U}}^n \widetilde{\mathbf{S}}^n \left[\widetilde{\mathbf{V}}^{(1)^n}, \widetilde{\mathbf{V}}^{(0)^n}\right], \tag{36}$$

we compute the equivalent channel matrix of the two nodes Sv and Sw, $\mathbf{H}^{(\mathrm{eq})^n} \in \mathbb{C}^{2\times(N-\tilde{r})} = \mathbf{H}_{vw_n}^{\mathrm{T}} \widetilde{\mathbf{V}}^{(0)^n}$, which assures that the interference signals from the other $N-2$ nodes are suppressed. The matrix $\widetilde{\mathbf{V}}^{(0)^n} \in \mathbb{C}^{M\times(N-\tilde{r})}$ contains the right singular vectors of $\widetilde{\mathbf{H}}_{vw_n}^{\mathrm{T}}$, with $\tilde{r}$ denoting the rank of matrix $\widetilde{\mathbf{H}}_{vw_n}^{\mathrm{T}}$.

5.3.1. Matched Filter. Having the equivalent channel for the two data streams to be superposed, for Matched Filter (MF), we first perform a receive matched filtering to improve the received signal level. Afterwards, we superpose both data streams by simply adding both matched filtered signals which can be expressed by multiplying the matched filtered signals with a vector of ones. Thus, the MF receive beamforming can be written as

$$\mathbf{m}_{\mathrm{Rc}}^n = \mathbf{H}^{(\mathrm{eq})^{n\,\mathrm{H}}} \mathbf{1}_2 \tag{37}$$

with $\mathbf{1}_2 = [1, 1]^{\mathrm{T}}$.

In order to transmit to all nodes, we need single-group multicast beamforming. Low complexity transmit beamforming algorithms for single-group multicast are treated in [33]. It is shown in [33] that the MF outperforms other linear single-group multicast transmit beamforming, for example, ZF and MMSE. Therefore, we consider the MF for the transmit beamforming given by

$$\mathbf{m}_{\mathrm{Tx}} = \mathbf{H}^* \mathbf{1}_N. \tag{38}$$

5.3.2. Semidefinite Relaxation. Since in multi-way relaying all nodes want to communicate with each other, we propose a fair transceive beamforming, Semidefinite Relaxation (SDR). The receive beamforming of SDR tries to balance the signal to noise ratios (SNRs) between the two nodes whose data streams are going to be superposed. Therefore, we need to maximise the minimum SNR between the two nodes based on the equivalent channel. This optimisation problem can be written as

$$\begin{aligned} &\max_{\mathbf{m}_{\mathrm{Rc}}^n} \quad \min_{i\in\{v,w\}} \left\{ \left| \frac{\mathbf{m}_{\mathrm{Rc}}^n \mathbf{h}_i^{(\mathrm{eq})^n}}{\sigma_{z_{\mathrm{RS}}}^2} \right|^2 \right\} \\ &\text{s.t.} \quad \|\mathbf{m}_{\mathrm{Rc}}^n\|_2^2 \le 1, \end{aligned} \tag{39}$$

which leads to a fair receive beamforming with $\mathbf{h}_i^{(\mathrm{eq})^n}$ being the equivalent channel of node Si whose data stream is going to be superposed. Such an optimisation problem is proved to be NP-hard in [34]. Nonetheless, such nonconvex quadratically constrained quadratic program can be approximately solved using SDR techniques. Some works have used SDR techniques for approximately solving max-min SNR problems, for example, [34] for single-group multicast and [35] for multigroup multicast, where [34] is a special case of [35] when the number of groups is one. As in [34], we rewrite the problem into a semidefinite program and make a relaxation by dropping the rank-one constraint. As a consequence, the solution might be higher rank [34]. However, good approximate solutions can be obtained using randomisation techniques as in [34]. Bounds on the approximation error of the SDR techniques have been developed in [36], which was motivated by the work in [34]. Having $\mathbf{X} = \mathbf{m}_{\mathrm{Rc}}^{n\ \mathrm{H}} \mathbf{m}_{\mathrm{Rc}}^n$ and $\mathbf{Q}_i = \mathbf{h}_i^{(\mathrm{eq})^n} \mathbf{h}_i^{(\mathrm{eq})^{n\,\mathrm{H}}} / \sigma_{z_{\mathrm{RS}}}^2$, and using semidefinite relaxation, we can rewrite (39) into

$$\begin{aligned} &\max_{\mathbf{X}} \quad \min_{i\in\{v,w\}} \operatorname{tr}\{(\mathbf{X}\mathbf{Q}_i)\} \\ &\text{s.t} \quad \operatorname{tr}\{\mathbf{X}\} = 1, \\ &\qquad \mathbf{X} \succeq \mathbf{0}. \end{aligned} \tag{40}$$

After introducing slack variables and rewriting (40) as in [34], we find the approximate solution of (39) using SeDuMi [37].

For SDR transmit beamforming, we consider a fair transmit beamforming which solves the optimisation problem of maximising the minimum SNR of

$$\begin{aligned} &\max_{\mathbf{m}_{\mathrm{Tx}}} \quad \min_{k\in\mathcal{I}} \left\{ \left| \frac{\mathbf{m}_{\mathrm{Tx}} \mathbf{h}_k^{\mathrm{T}}}{\sigma_{z_k}^2} \right|^2 \right\} \\ &\text{s.t.} \quad \|\mathbf{m}_{\mathrm{Tx}}\|_2^2 \le 1. \end{aligned} \tag{41}$$

Similar to (39), (41) can be approximately solved with semidefinite relaxation techniques using a solver such as SeDuMi [37].

As mentioned before, the n-th BC phase ANCTB is decoupled into receive beamforming and transmit beamforming. The ANCTB receive beamforming matrix in the n-th phase is given by

$$\mathbf{G}_{\text{Rc}}^{n} = \left[\tilde{\mathbf{V}}^{(0)^{n}} \mathbf{m}_{\text{Rc}}^{n}\right]^{\text{T}}, \tag{42}$$

and the ANCTB transmit beamforming in the n-th phase is given by

$$\mathbf{G}_{\text{Tx}} = [\mathbf{m}_{\text{Tx}}]\boldsymbol{\Gamma}^{1/2} \tag{43}$$

with the power loading matrix $\boldsymbol{\Gamma} \in \mathbb{R}_{+}$ given by

$$\boldsymbol{\Gamma} = \left(\text{mean}\left(|\mathbf{H}^{\text{T}}\mathbf{m}_{\text{Tx}}|\right)\right)^{-1}, \tag{44}$$

where the modulus operator $|\cdot|$ is assumed to be applied element wise and the mean function returns the mean of a vector. In order to satisfy the transmit power constraint at the RS, a normalisation factor $\beta \in \mathbb{R}_{+}$ is needed with

$$\beta = \sqrt{\frac{q_{\text{RS}}}{\operatorname{tr}\left(\mathbf{G}_{\text{Tx}}\mathbf{G}_{\text{Rc}}^{n}\left(\mathbf{H}\mathbf{R}_{\mathbf{x}}\mathbf{H}^{\text{H}} + \mathbf{R}_{\mathbf{z}_{\text{RS}}}\right)\mathbf{G}_{\text{Rc}}^{n^{\text{H}}}\mathbf{G}_{\text{Tx}}^{\text{H}}\right)}}. \tag{45}$$

Finally, the ANCTB is given by

$$\mathbf{G}^{n} = \beta\mathbf{G}_{\text{Tx}}\mathbf{G}_{\text{Rc}}^{n}. \tag{46}$$

6. Performance Analysis

In this section, we analyse the sum rate performance of non-regenerative multi-way relaying in a scenario where $N = 3$ single-antenna nodes communicate to each other with the help of a non-regenerative RS with $M = 3$ antenna elements. We set $q_{\text{RS}} = 1, \sigma_{z_{\text{RS}}}^2 = \sigma_{z_k}^2 = 1$, for all $k, k \in \mathcal{I}$ and $\sigma_x^2 = 1$. We use an i.i.d. Rayleigh channel and set the SNR equal to the channel gain. We assume reciprocal and stationary channels within N communication phases. We start by analysing the case of multiplexing transmission with SMTB for the symmetric and asymmetric traffic cases. We then compare the analog network coded transmission with multiplexing transmission for the case of asymmetric traffic.

Figure 3 shows the sum rate performance for the symmetric traffic case of multiplexing transmission with SMTB as a function of SNR in dB. MMSE outperforms ZF and MSNR as expected. However, to compute the transmit beamforming, MMSE needs the information of the noise variance at the nodes which increases the signaling effort in the network. In the high-SNR region, ZF converges to MMSE, while, in the low-SNR region, MSNR converges to MMSE. If the RS applies ZF transceive beamforming, there is no performance improvement even if the nodes apply interference cancellation. This is due to the fact that the interference has been canceled already at the RS. MMSE is able to obtain a slight performance improvement if interference cancellation is applied at the nodes. The highest

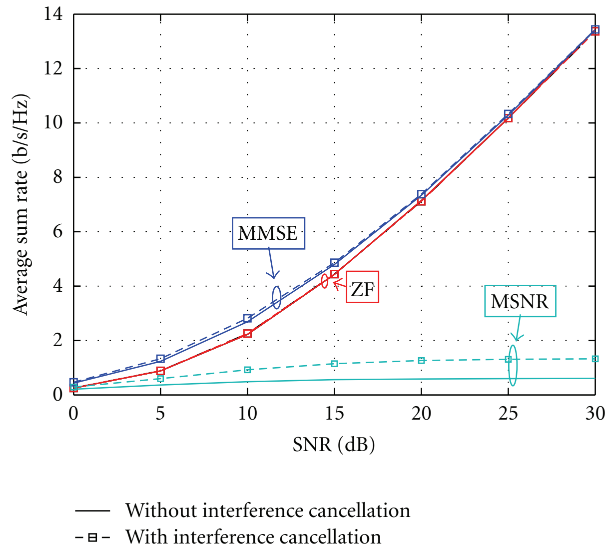

Figure 3: Sum rate performance of three-way relaying for multiplexing transmission with SMTB and symmetric traffic.

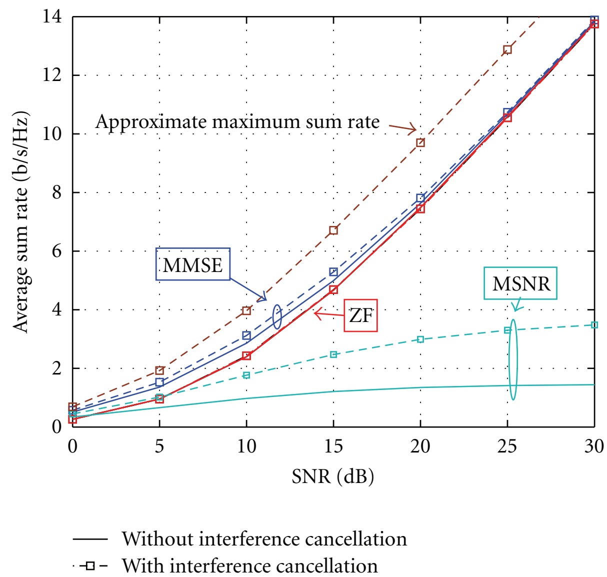

Figure 4: Sum rate performance of three-way relaying for multiplexing transmission with SMTB and asymmetric traffic.

performance improvement due to interference cancellation at the nodes is obtained when the RS uses MSNR. MSNR does not manage the interference, thus, if the nodes are able to perform interference cancellation, the performance is significantly improved.

Figure 4 shows the sum rate performance for the asymmetric traffic case of multiplexing transmission with SMTB. It can be seen that the sum rate performance is higher than in the symmetric traffic case. This is due to the fact that in the symmetric traffic we take the worst link as the one which defines the overall rate. Once again, as expected,

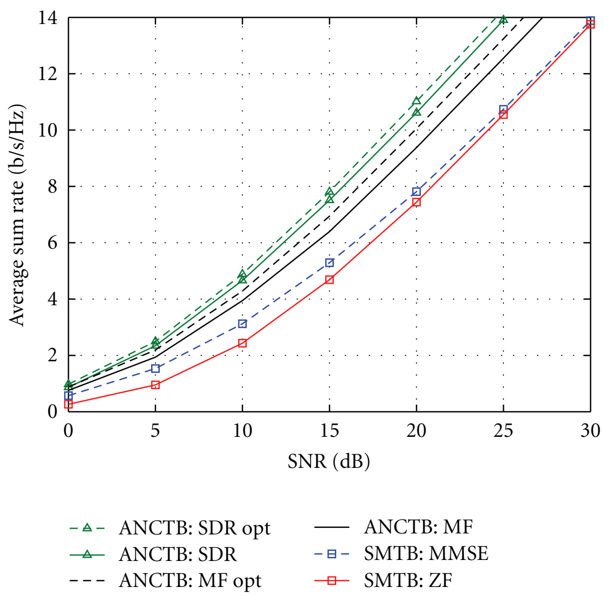

Figure 5: Sum rate performance of three-way relaying with asymmetric traffic: SMTB versus ANCTB.

MMSE performs the best and ZF converges to MMSE in the high-SNR region and MSNR converges to MMSE in the low-SNR region. The performance gain for both MMSE and MSNR when the nodes apply interference cancellation is higher than in symmetric traffic case. Furthermore, a curve termed approximate maximum sum rate is shown in Figure 4. For that curve, the maximisation of the sum rate in (21) is solved numerically using fmincon from MATLAB to provide an approximated maximum sum rate of multiplexing transmission. Since the problem in (21) is nonconvex, fmincon only guarantees a locally optimum solution. Moreover, the solution depends on the chosen starting point. In this paper, we use the values of MMSE transceive beamforming as the starting point. As can be seen, there is a gap between the approximated maximum sum rate and the suboptimum transceive beamforming algorithms. Despite the performance gap, the suboptimum transceive beamforming algorithms are easier to be implemented, and thus, are practically interesting.

Figure 5 shows the sum rate performance comparison of multiplexing transmission and analog network coded transmission for the asymmetric traffic case. It can be seen that the analog network coded transmission with ANCTB outperforms multiplexing transmission with SMTB, which shows the benefit of beamforming-based PLNC for non-regenerative multi-way relaying. The ANCTB SDR outperforms ANCTB MF with the penalty of having higher computational complexity to find the solution of the optimisation problem. Moreover, ANCTB SDR needs feedback channels to obtain the information of the noise variance of the nodes to compute the transmit beamforming.

In this paper, we propose a method to superpose two data streams out of N data streams which does not need any signaling in the network. The corresponding curves are indicated by ANCTB: MF and ANCTB: SDR. In Section 3, we addressed the general rule for the superposition of the two data streams for analog network coded transmission. We also provided the possible superposition options for $N = 3$. In Figure 5, we provide the curves ANCTB: MF opt and ANCTB: SDR opt, where the RS searches the optimum superposition among all possible options. It can be seen that, in the case of $N = 3$, the performance of the proposed suboptimum superposition method is not far away from the optimum one, especially in the case of fair transceive beamforming ANCTB-SDR and/or in the low-SNR region. Therefore, the suboptimum method offers a good trade off between the performance and the required signaling in the network.

In this paper, we assume that $M \geq N$ and an i.i.d. channel, and, therefore, the proposed suboptimum algorithms works well. If $M < N$ and/or when there are channel correlations, one can expect a performance degradation. We also assume that perfect CSI is available so that the RS is able to perform transceive beamforming. However, in order to obtain the CSI, there are additional resources needed for the RS and the nodes to estimate the channels. It is still an open issue on how to obtain the CSI at the RS and at all the nodes for non-regenerative multi-way relaying. One approach that can be used is to extend the channel estimation methods for non-regenerative two-way relaying in [38, 39].

7. Conclusion

In this paper, we propose non-regenerative multi-way relaying where a multi-antenna non-regenerative RS assists N nodes to communicate to each other. The number of communication phases is equal to the number of nodes, N. Two transmission methods are proposed to be applied at the RS, namely, multiplexing transmission and analog network coded transmission. Optimum transceive beamforming maximising the sum rate is addressed. Due to the nonconvexity of the optimisation problem, suboptimum but practical transceive beamforming are proposed, namely, ZF, MMSE, and MSNR for multiplexing transmission, and MF and SDR for analog network coded transmission. It is shown that analog network coded transmission with ANCTB outperforms multiplexing transmission with SMTB, which shows the benefit of beamforming-based PLNC for non-regenerative multi-way relaying.

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